An identity relating Eisenstein series on general linear groups

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Kernel integrals

cusp form

$$\mathcal{I}(f_s,\varphi_\pi)(\underline{h}) = \int\limits_{G(k)\backslash G(\mathbb{A})} \varphi_\pi(g) \underbrace{K(f_s)(\overbrace{t(g,h)}^{\text{Embedding in } \underline{H}})}_{\text{kernel function}}\, dg.$$

cuspidal rep. of G

kernel function: Eisenstein series or Fourier coef. of Eisenstein

choose: H big group
t(g,h) embedding
K kernel function

D. Ginzburg and D. Soudry. *Integrals derived from the doubling method*. IMRN (2020).

degenerate

O_{2+2m}

A construction

$$\mathrm{E}\left(f_{\chi,s},\varphi_\sigma\right)(h)=\int\limits_{\mathrm{O}_{2m}(k)\backslash \mathrm{O}_{2m}(\mathbb{A})}\varphi_\sigma(g)E\left(f_{\chi,s}\right)(t(g,h))\,dg.$$

$g=\begin{pmatrix} a & b \\ c & d\end{pmatrix}\in O_{2m}(\mathbb{A})$

$t(g,h)=\begin{pmatrix} a & & b \\ & h & \\ c & & d\end{pmatrix}$

cuspidal rep of O_{2m}

$\mathrm{Ind}_{Q_{2m+1}(\mathbb{A})}^{O_{2(2m+1)}(\mathbb{A})}\chi(\det)|\det|^s$

Eisenstein series

$\mathrm{Ind}_{Q_1(\mathbb{A})}^{O_{2+2m}(\mathbb{A})}\chi|\cdot|^s\otimes\sigma$

A construction

$$\mathrm{E}\left(f_{\chi,s},\varphi_\sigma\right)(h)=\int\limits_{\mathrm{O}_{2m}(k)\backslash\mathrm{O}_{2m}(\mathbb{A})}\varphi_\sigma(g)E\left(f_{\chi,s}\right)(t(g,h))\,dg.$$

Theorem [Ginzburg–P-S–Rallis, 1997]

$\mathrm{E}\left(f_{\chi,s},\varphi_\sigma\right)$ is an Eisenstein series.

Unramified calculations

fix Weil element

Local integrals arise from the global integral

$$I\left(f_{\chi_\nu,s},\xi_{\sigma_\nu}\right)(h)=\int\limits_{O_{2m}(k_\nu)} f_{\chi_\nu,s}\left(\tilde{\varepsilon}t\underline{(h,g)}\right)\underline{\chi_\nu^{-1}(\det(g))\sigma_\nu(g)}\,\xi_{\sigma_\nu}dg.$$

Sectio

vector in V_σ

Unramified calculations

Local integrals arise from the global integral

$$I\left(f_{\chi_\nu,s},\xi_{\sigma_\nu}\right)(h)=\int\limits_{O_{2m}(k_\nu)} f_{\chi_\nu,s}\left(\tilde{\varepsilon}t(h,g)\right)\chi_\nu^{-1}(\det(g))\sigma_\nu(g)\,\xi_{\sigma_\nu}dg.$$

Theorem [Ginzburg–P-S–Rallis, 1997]

$$I\left(f^\circ_{\chi_\nu,s},\xi^\circ_{\sigma_\nu}\right)\left(I_{2+2m}\right)=\frac{L\left(s+1,\chi_\nu\otimes\sigma_\nu\right)}{\prod_{j=1}^m L\left(2s+2j,\chi_\nu^2\right)}\xi^\circ_{\sigma_\nu}.$$

$$\mathcal{E}(f_s,\varphi_\pi)(h) = \int_{Z_n(\mathbb{A})GL_n(k)\backslash GL_n(\mathbb{A})} \varphi_\pi(g)\, E_{mn}(f_s)(t(h,g))\, dg$$

kronecker - product

E_{mn} $\downarrow$ GL_n

$\mathrm{Ind}_{P_{mn-1,1}(\mathbb{A})}^{GL_{mn}(\mathbb{A})} \delta_{P_{mn-1,1}}^{s-\frac{1}{2}}$ $h \in GL_m$

I applied this philosophy for GL_n.

$L(\pi, \text{-''-})$

matrix coef.

Theorem [H., 2022]

$$I\left(f^{\circ}_{s,\nu}, \xi^{\circ}_{\pi_\nu}\right)(I_m) = \frac{Z_{\mathrm{GJ}}\left(m(s+\frac{1}{2}) - \frac{n-1}{2}, c_{\xi^{\circ}_{\pi_\nu}, \check{\xi}^{\circ}_{\pi_\nu}}, \Phi_0\right)}{L\left(m(s+\frac{1}{2}), \omega_{\pi_\nu}\right)} \xi^{\circ}_{\pi_\nu}.$$

indicator of

$M_n(\mathcal{O})$

There are exactly n double cosets in $P_{mn-1,1}\backslash\mathrm{GL}_{mn}/t\left(\mathrm{GL}_m \times \mathrm{GL}_n\right)$

For $0 \leq r \leq n-1$

$$\varepsilon_r := \begin{pmatrix} I_{(m-r)n-1} & & \\ & & I_{rn} \\ & 1 & \end{pmatrix} \begin{pmatrix} I_{(m-r)n-1} & 0 & 0 \\ & 1 & \underline{b_r} \\ & & I_{rn} \end{pmatrix},$$

where $\underline{b_r} := \left(e_{n-1}^T, e_{n-2}^T, \ldots, e_{n-r}^T\right)$.

Thank You!