35TH AUTOMORPHIC FORMS WORKSHOP

TITLES AND ABSTRACTS

- Speaker: Kevin Allen (University College Dublin)
  Title: Unimodal sequences, Hecke-type double sums and false theta series
  Abstract: The study of strongly unimodal sequences has recently attracted considerable interest with connections to knot theory and (mixed) mock modularity. In this talk, we discuss a two-parameter generalization of a Hecke-Appell type expansion for the generating function of unimodal sequences and its connection to Hecke-type double sums and false theta series. This is joint work in progress with Robert Osburn (UCD).
- Speaker: Michael Allen (Louisiana State University)
  Title: Holomorphic projection of sesqui-harmonic Maass forms
  Abstract: The theory of holomorphic projections has been used to prove a number of results in the setting of mock modular and harmonic Maass forms. As a motivating example, we consider Zagier's weight $3/2$ mock Eisenstein series

$$\mathcal{H}^{+} = \frac{-1}{12} + \sum_{n\geq 1} H(n)q^{n},$$

  where the $H(n)$ are Hurwitz class numbers. Using the holomorphic projection of the corresponding Maass form, Beckwith, Raum, and Richter recently obtained Ramanujan-type congruences for Hurwitz class numbers.
  Duke, Imamo = glu, and Tóth consider a generating function whose coefficients relate to real quadratic fields in an analogous manner to the relationship between Hurwitz class numbers and imaginary quadratic fields. Specifically, they show that this function can be completed to a sesqui-harmonic Maass form—a real analytic function which transforms like a modular form and whose shadow is a harmonic Maass form. Motivated by this example, we investigate the theory of holomorphic projection for sesqui-harmonic forms and what they can tell us about the coefficients of Duke, Imamo = glu and Tóth's function. This is joint work in progress with Olivia Beckwith and Vaishavi Sharma.
- Speaker: Mondal Archita (IIT Bombay) (joint work with R. Preeti)
  Title: Rational equivalence of adjoint classical groups of type Dn over fields of virtual cohomological dimension two
  Abstract: Let F be a field of characteristic different from 2 and such that virtual cohomological dimension of F is 2. Let G be a semisimple classical adjoint group of type D_n defined over F. We show that $G(F)/R = 0$, where R denotes rational equivalence on $G(F)$. The analogous result for groups of type ${}^1A_n$ and B_n has been proved by Merkurjev, for groups of type ${}^2A_{2n}$ by Voskresenskii-Klyachko and for general groups of type ${}^2A_n$ and C_n by Kulshreshta-Parimala. Combining the main theorem of this paper with the above mentioned results, we have $G(F)/R$ is trivial, for any semisimple adjoint classical group G defined over F.
- Speaker: Walter Bridges (University of Cologne)
  Title:
  Unimodal Sequences and Modularity
  Abstract:

Date: May 2023.

Abstract: A unimodal sequence of positive integers is like a two-sided integer partition, where parts may increase to a peak then decrease. In contrast to the often modular and mock modular generating functions one obtains for partitions, unimodal sequences are connected to false theta functions. Thanks to recent work of Bringmann and Nazaroglu, such functions can be completed à la Zwegers to real analytic Jacobi forms. We extend their work to prove false Jacobi transformations for functions that keep track of the rank statistic, and we develop a method to prove precise asymptotic series for the moments of such statistics. We also announce a proof of the log-concavity for unimodal sequences. This is joint work with Kathrin Bringmann.

- Speaker: Benjamin Brindle (University of Cologne)

  Title: Asymptotic expansions for partitions generated by infinite products

  Abstract: We study partitions with parts in $\mathcal{L} \subset \mathbb{N}$ $(\gcd(\mathcal{L}) = 1)$. and good analytic properties of the corresponding zeta function. Using the circle method, one can prove asymptotic formulas if $\mathcal{L}$ is a multiset of integers and the zeta function has multiple poles. This is j.w.w. W. Bridges, K. Bringmann, and J. Franke.

- Speaker:Alejandro De Las Penas Castano (University of Virginia)

  Title: Inversion Formulas for the modular j-function.

  Abstract: The modular j-function is a bijection between the fundamental domain of the action of $\mathrm{SL}_2(\mathbb{Z})$ on the upper half plane and complex plane and is thus an invertible function. One might naturally ask about what form does this inverse function might take. One approach was given recently by Hong, Mertens, Ono, and Zhang that expresses the Taylor series of the modular j-function around the elliptic points i and $\rho = e^{\pi i/3}$ as rational functions arising from the signature 2 and 3 cases of Ramanujan's theory of elliptic functions to alternative bases. In joint work with Badri Vishal Pandey, we extend these results and give inversion formulas for the j-function around i and ρ arising from Gauss' hypergeometric functions and Ramanujan's theory in signatures 4 and 6.

- Speaker: Archer Clayton (Brigham Young University)

  Title: The Effect of the Trace Operator on the Duality of Modular Grids in Genus Zero

  Abstract: Griffin, Jenkins, and Molnar showed that given a space of weakly holomorphic modular forms, there is another space of weakly holomorphic modular forms which is dual to the original space. When this occurs, the coefficients of the Fourier expansions of the elements of a canonical basis appear as negatives of Fourier coefficients for a canonical basis of the other space. We investigate the effect of the trace operator on this duality in spaces of genus zero.

- Speaker: Jonathan Cohen (University of North Texas)

  Title: Fixed vectors in depth-zero supercuspidal representations of GSp(4).

  Abstract: We will discuss an approach to compute dimensions of certain spaces of classical Siegel modular forms. This leads to the question of dimensions of fixed vectors under certain sequences of congruence subgroups in representations of p-adic GSp(4). For general representations, these dimensions remain unknown. We will illustrate the situation in the special case of depth-zero supercuspidal representations; these include the first examples of (generic) representations for which the fixed-vector dimensions have been computed.

- Speaker: Will Craig (University of Cologne)

  Title: Zeros of period polynomials of cusp forms

  Abstract:

  We consider the period polynomials $r_f(z)$ associated with cusp forms of weight k on all of $\mathrm{SL}_2(\mathbb{Z})$, which are generating functions for the critical L-values of the modular L-function associated to f. In 2014, El-Guindy and Raji proved that if f is an eigenform, then $r_f(z)$ satisfies a "Riemann hypothesis" in the sense that all its zeros lie on the natural boundary of its functional equation. We show that this phenomenon is not restricted to eigenforms, and we provide natural infinite families of cusp forms whose period polynomials almost always satisfy the Riemann hypothesis. In particular, we show that for weights $k \geq 180$, a positive proportion of cusp forms f have period polynomials which satisfy the Riemann hypothesis.

- Speaker: Agniva Dasgupta (Texas A&M University)
  Title: Second Moment of Twisted Cusp Forms Along a Coset
  Abstract:
  Abstract: We prove Lindelöf-on-average upper bound for the second moment of the L-function associated to a level 1 holomorphic cusp form, twisted along a coset of subgroup of the characters modulo $q^{2/3}$ (where $q = p^3$ for some odd prime p). This result should be seen as a q-aspect analogue of Anton Good's (1982) result on upper bounds of the second moment of cusp forms in short intervals.
- Speaker: Tim Davis (Queen Mary, University of London)
  Title: p-adic valuation of local Whittaker newforms
  Abstract: In this talk we discuss the problem of providing good lower bounds for p-adic valuation of local Whittaker newforms with non-trivial central character. We will then give an outline how this local problem can give insight into the global problem of finding p-adic valuation of the Fourier coefficients of modular forms at cusps. This generalises a result of Cesnavicius, Neururer and Saha who consider the setting of trivial central character.
- Speaker: Thomas Driscoll-Spittler (TU Darmstadt)
  Title:
  Reflective automorphic forms of singular weight and their expansions at 1-dimensional cusps
  Abstract:
  We study the classification of reflective automorphic products of singular weight on regular lattices. We show that there are exactly 11 such automorphic forms on lattices splitting two hyperbolic planes. Their expansions at suitable 1-dimensional cusps yield the 70 affine structures found by Schellekens in his classification of holomorphic vertex operator algebras of central charge 24. This is joint work with Nils Scheithauer and Janik Wilhelm.
- Speaker: Melissa Emory (Oklahoma State University)
  Title: Beyond Endoscopy via Poisson Summation
  Abstract: Langlands proposed a strategy called Beyond Endoscopy to prove the principle of functoriality, which is one of the central questions of present day mathematics. A first step was achieved by Ali Altug who worked with the group $\mathrm{GL}(2)$ over the rationals. This project generalizes Altug's result to a number field. In this talk we will emphasize some interesting differences between our work and Altug's work.
- Speaker: Jack Fogliasso (Wayne State University)
  Title: Jacobi Forms and Wronskians
  Abstract: In 2003, G. Mason used a modular version of the Wronskian from classical ODE theory to study vector-valued modular forms. We explore an analogue of the Wronskian for Jacobi forms. Since Jacobi forms are functions of two complex variables, there are two partial derivatives to consider. We characterize the transformation properties (with respect to the standard Jacobi group slash action) of mixed partial derivatives of arbitrary order. Generically, the Wronskian of Jacobi forms is not a Jacobi form but a quasi-Jacobi form, which has an additional parameter called its depth. We are interested in finding the depth of an arbitrary Wronskian of Jacobi forms. This is an ongoing research project with Prof. Matthew Krauel (Sacramento State).
- Speaker: Soumendra Ganguly (Texas A&M University)
  Title: Subconvexity for twisted L-functions on GL(3) $\times$ GL(2) and GL(3)
  Abstract: Let ϕ be the symmetric-square lift of an $\mathrm{SL}(2, \mathbb{Z})$ Hecke-Maass form. Let q be an odd cube-free positive integer, and let χ be a primitive Dirichlet character modulo q such that χ is not quadratic. Let f be an even Hecke-normalized Hecke-Maass newform of level dividing q and central character $\overline{\chi}^2$. We show q-aspect subconvexity bounds for $L(1/2, \phi \times f \times \chi)$ and $L(1/2+it), \phi \times \chi)$.
- Speaker: Brian Grove (Louisiana State University)

Title: Hypergeometric Moments Via Hecke Trace Formulas

Abstract: Abstract: The classical Sato-Tate problem tells us the distribution of normalized trace of Frobenius values for a fixed non-CM elliptic curve over $\mathbb{Q}$ converges to a semi-circular distribution as p tends to infinity. Recently, Ono, Saad, and Saikia found the distribution is also semi-circular if we now look at all elliptic curves over $\mathbb{Q}$ in the Legendre family. Along the way, the authors define a hypergeometric moment, which can be viewed as an average of the normalized trace of Frobenius values as p tends to infinity, and establish a few cases. I will describe how to use trace formulas for modular forms on specific congruence subgroups to prove additional cases of hypergeometric moments.

- Speaker: Pam Gu (Duke University)

  Title: A family of period integrals related to triple product L-functions

  Abstract: Let F be a number field with ring of adeles $\mathbb{A}_F$. Let r_1, r_2, r_3 be a triple of positive integers and let $\pi := \otimes_{i=1}^{3} \pi_i$ where the π_i are all cuspidal automorphic representations of $\mathrm{GL}_{r_i}(\mathbb{A}_F)$. We denote by $L(s, \pi, \otimes^3) = L(s, \pi_1 \times \pi_2 \times \pi_3)$ the corresponding triple product L-function. It is the Langlands L-function defined by the tensor product representation $\otimes^3$: ${}^L(\mathrm{GL}_{r_1} \times \mathrm{GL}_{r_2} \times \mathrm{GL}_{r_3}) \longrightarrow \mathrm{GL}_{r_1 r_2 r_3}(\mathbb{C})$. In this talk I will present a family of Eulerian period integrals, which are holomorphic multiples of the triple product -function in a domain that nontrivially intersects the critical strip. We expect that they satisfy a local multiplicity one statement and a local functional equation. This is joint work with Jayce Getz, Chun-Hsien Hsu and Spencer Leslie.

- Speaker: Rajat Gupta (University of Texas at Tyler)

  Title: Identities Associated with Dirichlet Series Satyisfying Hecke's Functional Equation

  Abstract: In this talk, we consider two sequences $a(n)$ and $b(n)$, $1 \leq n < \infty$, generated by

$$\sum_{n=1}^{\infty} \frac{a(n)}{\lambda_n^s} \quad \text{and} \quad \sum_{n=1}^{\infty} \frac{b(n)}{\mu_n^s}.$$

satisfying a familiar functional equation involving the gamma function $\Gamma(s)$. We will establish two general identities which can be thought of as a 'modular' or 'theta' relation wherein modified Bessel functions, instead of exponential functions, appear.

The arithmetical functions appearing in the identities include the Fourier coefficients a_n of a cusp form $f(z) = \sum_{n=0}^{\infty} a_n q^n$ in $S_k(\Gamma)$, where $\Gamma = \mathrm{SL}_2(\mathbb{Z})$, Ramanujan's arithmetical function $\tau(n)$; the number of representations of n as a sum of k squares $r_k(n)$; and primitive Dirichlet characters $\chi(n)$. If time permits we will also discuss some other important arithmetical functions.

This is a recent joint work with Professor Bruce C. Berndt, Professor Atul Dixit, and Professor Alexandru Zaharescu.

- Speaker: Markos Karameris (Technion-Israel Institute of Technology)

  Title: Hecke Algebras and Newspaces of Forms with Non-Trivial Character

  Abstract: Let $\mathcal{S}_k(\Gamma_0(N), \chi)$ denote the space of cuspforms with Dirichlet character χ and modular subgroup $\Gamma_0(N)$. We characterize the newspace $\mathcal{S}_k^{new}(\Gamma_0(N), \chi)$ as the intersection of eigenspaces of a particular family of Hecke operators generalizing the work of Baruch-Purkait [2015] to forms with non-trivial character. To do that we explicitly describe the Hecke algebra of locally constant compactly supported functions $\mathcal{H}(GL_2(\mathbb{Z}_p)//K_0(p^n), \chi_{id})$ where χ_{id} is an idelic character and $K_0(p^n)$ the Iwahori subgroup of level n. We then obtain representation theoretic results for this Hecke algebra which we de-adelizize into relations of classical operators.

- Speaker: Matija Kazalicki (University of Zagreb)

  Title: Second moments and the bias conjecture for the family of cubic pencils

  Abstract: For a 1-parametric family $\mathcal{F}_k$ of elliptic curves over $\mathbb{Q}$ and a prime p, consider the second moment sum $M_{2,p}(\mathcal{F}_k) = \sum_{k \in \mathbb{F}_p} a_{k,p}^2$, where $a_{k,p} = p + 1 - \#\mathcal{F}_k(\mathbb{F}_p)$. Inspired by Rosen and Silverman's proof of Nagao conjecture which relates the first moment of a rational elliptic surface to the rank of the Mordell-Weil group of the corresponding elliptic curve, S. J. Miller initiated the study of the asymptotic expansion of $M_{2,p}(\mathcal{F}_k) = p^2 + O(p^{3/2})$. He conjectured that the largest lower order term that does not average to 0 is on average negative (i.e. has a negative bias). In this paper, we provide an explicit formula for the second moment $M_{2,p}(\mathcal{F}_k)$ of

$$\mathcal{F}_k : y^2 = P(x)k + Q(x),$$

where $\deg P(x), \deg Q(x) \leq 3$. For a generic choice of polynomials $P(x)$ and $Q(x)$ this formula is expressed in terms of the point count of a certain genus two curve. Assuming the Sato-Tate conjecture for genus two curves, we prove that the Bias Conjecture holds for the pencil of the cubics $\mathcal{F}_k$. This is joint work with Bartosz Naskręcki.

- Speaker: Gene Kopp (Louisiana State University)
  Title: The Shintani-Faddeev modular cocycle
  Abstract: We ask the question, "how does the infinite q-Pochhammer symbol transform under modular transformations?" and connect the answer to that question to the Stark conjectures. The infinite q-Pochhammer symbol transforms by a generalized factor of automorphy, or modular 1-cocycle, that is analytic on a cut complex plane. This "Shintani-Faddeev modular cocycle" is an $\mathrm{SL}_2(\mathbb{Z})$-parametrized family of functions generalizing Shintani's double sine function and Faddeev's noncompact quantum dilogarithm. We relate real multiplication values of the Shintani-Faddeev modular cocycle to exponentials of certain derivative L-values, conjectured by Stark to be algebraic units generating abelian extensions of real quadratic fields.
- Speaker: Rahul Kumar (The Pennsylvania State University)
  Title: Applications of Lipschitz summation formula and asymptotic of plane partitions
  Abstract:
  Summation formulas have proven to be extremely useful in many areas of mathematics, including analysis and number theory. One such formula is the Lipschitz summation formula, which we will discuss in this talk. We will present a new transformation formula that generalizes Ramanujan's result, which itself a generalization of the modular transformation of Eisenstein series on $\mathrm{SL}_2(\mathbb{Z})$. Additionally, we will demonstrate another application of the Lipschitz summation formula by proving a non-modular transformation for $\sum_{n=1}^{\infty} \sigma_{2m}(n)e^{-ny}$. Using these results, we will derive a generalization of Wright's asymptotic estimate for the generating function of the number of plane partitions of a positive integer n. This talk is based on joint work with Professor Atul Dixit.
- Speaker: Eleanor McSpirit (University of Virginia)
  Title: Lattice Cohomology and Quantum Modular Forms
  Abstract: In 1999, Lawrence and Zagier established a connection between modular forms and the Witten-Reshetikhin-Turaev invariants of 3-manifolds by constructing q-series whose radial limits at roots of unity recover these invariants for particular manifolds. These q-series gave rise to some of the first examples of quantum modular forms. Using a 3-manifold invariant recently developed Akhmechet, Johnson, and Krushkal, one can obtain an infinite family of quantum modular invariants which realize the series of Lawrence and Zagier as a special case. This talk is based on joint work with Louisa Liles.
- Speaker: Pietro Mercuri (Sapienza Università di Roma)
  Title: Automorphism group of Cartan modular curves
  Abstract:
  We consider the modular curves associated to a Cartan subgroup of $\mathrm{GL}(2, \mathbb{Z}/n\mathbb{Z})$ or to a particular class of subgroups of $\mathrm{GL}(2, \mathbb{Z}/n\mathbb{Z})$ containing the Cartan subgroup as a normal subgroup. We describe the automorphism group of these curves when the level is large enough. If time permits, we give a sketch of the proof.
- Speaker: Shashika Petta Mestrige (Vermont Technical College)
  Title: Congruences for some partition functions and their ℓ-adic properties.
  Abstract:

Ramanujan, Watson, Atkin, Gordon, and Hughes used modular functions and modular equations to prove remarkable congruences of the partition function $p(n)$ and multi-partitions $p_k(n)$. By extending their ideas, we proved the congruences for two parameter family of partitions $p_{[1^c\ell^d]}(n)$ modulo powers of ℓ where ℓ is a prime $5 \leq \ell \leq 17$. We define these partitions by

$$\sum_{n=0}^{\infty} p_{[1^c\ell^d]}(n)q^n = \prod_{n=1}^{\infty} \frac{1}{(1-q^n)^c(1-q^{\ell n})^d}.$$

Then we used them to derive congruences and incongruences for ℓ-regular partitions, ℓ-core partitions, and ℓ-colored generalized Frobenius partitions.

Then, I will talk about the ℓ-adic modules related to $p_{[1^c\ell^d]}(n)$. Using these modules we were able to determine why the Ramanujan congruences only occur at certain primes for $p_{[1^c\ell^d]}(n)$ Our work has been motivated by the work of Boylan-Webb for the partition function $p(n)$.

- Speaker: Steven J. Miller (Williams College)

  Title: The Katz-Sarnak Density Conjecture and Bounding Central Point Vanishing of L-Functions

  Abstract:

  Montgomery and Dyson discovered in the 1970's that random matrix theory models spacings between zeros of L-functions away from the central point. These models are insensitive to finitely many zeros, and thus miss the behavior near the central point. This is the most arithmetically interesting place; for example, the Birch and Swinnerton Dyer conjecture states that the rank of the Mordell-Weil group equals the order of vanishing of the associated L-function there. To investigate the zeros near the central point, Katz and Sarnak developed a new statistic, the n-level density; one application is to bound the average order of vanishing at the central point for a given family of L-functions by an integral of a weight against some test function ϕ. While the 1-level density has been studied in prior work, larger n yield better bounds, but new technical problems emerge in the higher level densities. We discuss how to resolve these, and obtain the best upper bounds to date on (what is conjectured to be) 0, the probability of vanishing to order at least $r \geq 2$ for cuspidal newforms.

  This talk is joint with numerous REU students.

- Speaker: Andreas Mono (joint work with Kathrin Bringmann)

  Title: A modular framework of functions of Knopp

  Abstract:

  This talk presents the construction of a modular completion of a function introduced by Knopp 30 years ago in his paper Modular integrals and their Mellin transforms. His function is closely related to a term by term lift of Zagier's influential $f_{k,D}$ function under the Bol operator. We begin with a motivation of the topic, and summarize some background briefly. Afterwards, we outline our constructions, and discuss their naturality. We connect our result to the more recently introduced concept of locally harmonic Maaßforms by Bringmann, Kane, and Kohnen about 10 years ago. Finally, we conclude with some open questions and further directions. This is joint work with Kathrin Bringmann.

- Speaker: Manuel Müller (TU Darmstadt)

  Title: The basis problem for modular forms for the Weil representation

  Abstract: Let L be an even positive definite lattice of even rank $m \in 2\mathbb{N}$ and let L' be its dual lattice. Let $k \in \mathbb{N}$ with $k \geq m/2$ and let P be a harmonic polynomial of degree $k - m/2$. Then the vector valued theta function $\theta_{L,P}$ associated to L and weighted with P is a modular form for $\mathrm{SL}_2(\mathbb{Z})$ of weight k with respect to the Weil representation $\rho_{L'/L}$, where L'/L is called the discriminant form associated to L. Two positive definite lattices are in the same genus if they have equal ranks and isomorphic discriminant forms. We show that if the rank m of L is large enough compared to the rank of L'/L, then the cusp forms of weight k for the Weil representation $\rho_{L'/L}$ are generated by the vector valued theta functions in the genus of L weighted with harmonic polynomials of degree $k - m/2$.

- Speaker: Badri Vishal Pandey (University of Cologne)

  Title: Linear congruence relations for exponents of Borcherds products

  Abstract: For all positive powers of primes $p \geq 5$, we prove the existence of infinitely many linear congruences between the exponents of twisted Borcherds products arising from a suitable scalar-valued weight $1/2$ weakly holomorphic modular form or a suitable vector-valued harmonic Maaßform. To this end, we work with the logarithmic derivatives of these twisted Borcherds products, and offer various numerical examples of non-trivial linear congruences between them modulo p=11. In the case of positive powers of primes $p = 2, 3$, we obtain similar results by multiplying the logarithmic derivative with a Hilbert class polynomial as well as a power of the modular discriminant function. Both results confirm a speculation by Ono. (joint work with Andreas Mono)
- Speaker: Wissam Raji (American University of Beirut)

  Title: On the non-vanishing of L-Functions in half-integer weight.

  Abstract: We show a non-vanishing result for the derivatives of L-functions associated with cuspidal Hecke eigenforms of half-integral weight in plus space.
- Speaker: Olav Richter (University of North Texas)

  Title: Congruences of Hurwitz class numbers

  Abstract: I will report on recent joint work with Olivia Beckwith and Martin Raum on congruences of Hurwitz class numbers. As an application we prove the existence of imaginary quadratic fields with class number not divisible by a given prime ℓ and with arbitrary splitting conditions at a finite number of primes. That application is in the spirit of results by Bhargava ($\ell = 3$) and Wiles (arbitrary ℓ), but our method is completely different.
- Speaker: Jenny Roberts (University of Bristol)

  Title: Newform Eisenstein congruences of local origin

  Abstract: The theory of Eisenstein congruences dates back to Ramanujan's surprising discovery that the Fourier coefficients of the discriminant function are congruent to the 11th power divisor sum modulo 691. This observation can be explained via the congruence of two modular forms of weight 12 and level 1; the discriminant function and the Eisenstein series, E_{12}. We explore a generalisation of this result to newforms of weight $k > 2$, squarefree level and non-trivial character
- Speaker: Hasan Saad (University of Virginia)

  Title: Determining point distributions on hypergeometric varieties

  Abstract: Automorphic forms are ubiquitous in mathematics. In this talk we discuss how harmonic Maass forms and holomorphic modular forms make it possible to determine the limiting distributions of point counts of certain families of hypergeometric varieties over finite fields. Namely, we obtain the $SU(2)$ for the family of Legendre elliptic curves and the $O(3)$ distribution for a family of $K3$ surfaces, with an explicit error term for the latter. We then show how to count "matrix" points on these varieties and determine the distributions of those counts.
- Speaker: Ali Saraeb (American University of Beirut)

  Title: The Generalizations of Rademacher's Method for the Transformation Law of the Jacobi Theta Function, ϑ_1

  Abstract: The four Jacobi functions played an important role in mathematical analysis and in number theory. Namely, one of the Jacobi functions appearsin the functional equation of the Riemann Zeta function and allows us to determine analytic continuation beyond the critical strip. In this talk, we present the generalization of a method employed by Rademacher who generalized Siegel's method to prove the transformation law of the Dedekind Eta function to prove the transformation law of one of the Jacobi functions. We, thus, aim at presenting a new proof inspired by Siegel and Rademacher for the transformation law of ϑ_1 under the action of the full modular group $\mathrm{SL2}(\mathbb{Z})$.
- Speaker: Pratiksha Shingavekar (Indian Institute of Technology, Madras)

  Title: 3-Selmer groups, ideal class groups and the rational cube sum problem

  Abstract:

Given an elliptic curve E over a number field F and an isogeny φ of E defined over F, the study of the φ-Selmer group has a copious history that can be traced back to the works of Cassels up to the recent works of Bhargava et al. and Chao Li. Let $E/\mathbb{Q}$ be an elliptic curve with a rational 3-isogeny φ. In this talk, we give an upper and a lower bound on the rank of the φ-Selmer group of E over $\mathbb{Q}(\sqrt{-3})$ in terms of the 3-part of the ideal class group of certain quadratic extension of $\mathbb{Q}(\sqrt{-3})$. Using our bounds on the Selmer groups, we prove some cases of Sylvester's conjecture on the rational cube sum problem. As an application of our bounds, we are able to produce an infinite family of elliptic curves having arbitrary large 3-Selmer rank over $\mathbb{Q}(\sqrt{-3})$. We also exhibit infinitely many imaginary quadratic fields and biquadratic fields with non-trivial 3-class groups. This is a joint work with Dr. Dipramit Majumdar and Dr. Somnath Jha.

- Speaker: Leah Sturman (Oregon State University)

  Title: On the Remaining Cases of Kang and Park's Conjecture

  Abstract: In 1956, Alder conjectured an integer partition inequality which generalized Euler's partition identity, the first Rogers-Ramanujan identity, and a partition identity of Schur. Alder's conjecture, proved in part by Andrews in 1971, followed by Yee in 2008, and finally completed by Alfes, Jameson, and Lemke Oliver in 2010 states that the number of d-distinct partitions of an integer n is at least the number of partitions of n into parts congruent to $\pm 1 \pmod{d+3}$. Whereas Andrews and Yee utilized combinatorial methods to prove the result for all but finitely many values of d, Alfes et al. used asymptotic methods to resolve the conjecture for the remaining cases. In 2020, Kang and Park conjectured a similar integer partition inequality which generalizes the second Rogers-Ramanujan identity. Duncan, Khunger, Swisher, and Tamura proved Kang and Park's conjecture for all but finitely many values of d using combinatorics. In this thesis, we use combinatorics to prove Kang and Park's conjecture for $d = 1$. We generalize the work of Alfes et al. to prove Kang and Park's conjecture for $d = 6$ and $8 \leq d \leq 61$. This is joint work with Holly Swisher.

- Speaker: Qihang Sun (UIUC)

  Uniform bound for half-integral weight sums of Kloosterman sums and the application on partitions.

  Abstract:

  A uniform and power-saving bound on the sum of generalized Kloosterman sums with the eta-multiplier, as the result by Ahlgren and Andersen in 2015, gives a better error estimate on the Rademacher exact formula of the partition function $p(n)$. We generalize their method to get a uniform bound corresponding to a wide class of half-integral multipliers and apply the bound on partitions of rank modulo 3.

- Speaker: Karen Taylor (Bronx Community College, CUNY)

  Title: Explicit Lift of Hyperbolic Poincaré Series to a Locally Harmonic Maass Form

  Abstract: In 2014, Bringmann, Kane, and Kohnen (BKK) introduced locally harmonic Maass forms. They gave an explicit lift of $f_{k,D}(\tau) = \sum_{\substack{a,b,c\in\mathbb{Z} \\ b^2-4ac=D}} \frac{1}{(a\tau^2+b\tau+c)^k}$, a hyperbolic Eisenstein series, to a locally harmonic Maass form. In this talk, we discuss BKK's construction and the question of generalizing to obtain the locally harmonic lift of hyperbolic Poincaré series. The definitions will be given in the talk. This is work in progress with Larry Rolen and Andreas Mono.

- Speaker: Jacqueline Voros (University of Bristol)

  Title: On the average least negative Hecke eigenvalue

  Abstract: Hecke eigenvalues, or Fourier coefficients of certain classical modular forms, hold more structure and information than they may seem. For instance, it is known that Fourier coefficients uniquely determine a cusp form (in relation to its weight and level). However, it has also been shown that to some extent just the signs of Fourier coefficients are needed to determine a unique cusp form. In this talk I investigate more properties of signs of these Fourier coefficients, and in particular where, on average, the first sign change is. We shall also see that this problem is strongly linked with an analogous problem on the least quadratic non-residue, and hence the intuition behind these results.

- Speaker: Clayton Williams (UIUC)

  Title: An Infinite Family of Vector-Valued Mock Theta Functions

  Abstract:

  We exhibit an infinite family of vector-valued mock theta functions indexed by positive integers coprime to 6. These are built from specializations of Dyson's rank generating function and related functions studied by Watson, Gordon, and McIntosh. The associated completed harmonic Maass forms transform according to the Weil representation attached to a rank one lattice. This strengthens a 2010 result of Bringmann and Ono and a 2019 result of Garvan. This is joint work with Nick Andersen of BYU.

- Speaker: Liyang Yang (Princeton University)

  Title: Relative Trace Formula and the Burgess Bound for Twisted L-functions

  Abstract: In this talk, we present a relative trace formula to establish a refined hybrid subconvex bound for $L(1/2, \pi \times \chi)$, where π is a unitary automorphic representation of $\mathrm{GL}(2)$ over a number field F, and χ is a Hecke character. Our approach leads to the Burgess subconvex bound, which can be expressed as follows:

  $$L(1/2, \pi \times \chi) \ll_{\pi,F,\varepsilon} C(\chi)^{\frac{1}{2}-\frac{1}{8}+\varepsilon},$$

  where $C(\chi)$ is the analytic conductor of χ.

- Speaker: Hui Xue (Clemson University)

  Title: Linear independence between even and odd periods

  Abstract: We investigate the linear independence between an even and an odd period of modular forms. We show that they are linearly independent in most cases.