A cusp form $f(z)$ of weight k for $\mathrm{SL}_2(\mathbb{Z})$ is determined uniquely by its first $\ell := \dim S_k$ Fourier coefficients. We derive an explicit bound on the nth coefficient of f in terms of its first ℓ coefficients. We use this result to study the non-negativity of the coefficients of the unique modular form of weight k with Fourier expansion

$$F_{0,k}(z) = 1 + O(q^{\ell+1}).$$

In particular, we show that $k = 81632$ is the largest weight for which all the coefficients of $F_{0,k}(z)$ are non-negative. This result has applications to the theory of extremal lattices. This is joint work with Jeremy Rouse.