

# EXPLICIT FORMULA AND RANKS OF ELLIPTIC CURVES

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ABSTRACT. A powerful tool in analytic number theory, the explicit formula relates the sum of zeros of an  $L$ -function to a sum over prime ideals. It has been applied to averaging the analytic ranks of a family of elliptic curves by Goldfeld, Brumer and Heath-Brown to prove, for example, that the average analytic rank over the twists of an elliptic curve over the rational numbers has rank less than  $3/2$ . It has been conjectured by Goldfeld that the rank should be, on average  $1/2$ .