KATSURADA'S CONJECTURE AND THE IKEDA IDEAL

JIM BROWN

ABSTRACT. Let f be an elliptic modular form of full level and weight $2k - n$ where n and k are even positive integers. If f is a normalized eigenform there is a construction due to Ikeda that associates to f a genus n Siegel eigenform $I_n(f)$ of weight $2k$. It turns out that a fairly natural question to ask is for which primes is $I_n(f)$ congruent to a Siegel modular form that does not arise as an Ikeda lift. Katsurada formulated a precise conjecture for exactly which primes there is such a congruence in terms of special values of the L-function of f and provided evidence for this conjecture.

In this talk I will define the Ikeda ideal of $I_n(f)$. This ideal measures all congruences between $I_n(f)$ and non-Ikeda lifts. In joint work with Rodney Keaton we provide a lower bound on the size of this ideal in terms of the same L-values as in Katsurada's conjecture. This result provides alternate evidence for Katsurada's conjecture as we have different (less restrictive) technical hypotheses on our result, but also retains the information on the powers of the primes dividing the L-values. Katsurada's conjecture only deals with if the prime divides the L-value or not.