

APÉRY'S DIFFERENTIAL EQUATIONS AND ELLIPTIC CURVES

MASHA VLASENKO

ABSTRACT. A proof of irrationality of $\zeta(3)$ by Roger Apéry used recursively given sequences of numbers whose integrality could be explained with the help of modular forms. We consider an ordinary differential equation of order 3 with several parameters which corresponds to recursions of a similar shape as in Apéry's case. To such an equation we relate an elliptic curve and show that if the values of parameters are such that the differential equation has modular parametrisation then the expansion of its analytic solution with respect to the natural parameter coincides, under certain assumptions, with the q -expansion of the newform of its spectral elliptic curve and therefore possesses a multiplicativity property. This technique allows us to get a list of parameters for which the differential equation is modular. The talk is based on my recent work with Vasily Golyshev.