L-function for $\mathrm{Sp}(4) \times \mathrm{GL}(2)$ via a non-unique model

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An Eisenstein series on GL_2

- Let F be a number field, $\mathbb{A}$ be its ring of adeles, and ψ a non-trivial additive character of $F\backslash\mathbb{A}$.
- We start with a construction of a GL_2 Eisenstein series. Let $\Phi \in \mathcal{S}(\mathbb{A}^2)$, which is a restricted tensor product $\mathcal{S}(\mathbb{A}^2) = \otimes'_\nu \mathcal{S}(F_\nu^2)$, where for $\nu|\infty$, $\mathcal{S}(F_\nu^2) = \mathcal{S}(\mathbb{R}^2)$ or $\mathcal{S}(\mathbb{C}^2)$ is the usual Schwartz space of infinitely differentiable and rapidly decreasing functions, and for $\nu < \infty$, $\mathcal{S}(F_\nu^2) = C_c^\infty(F_\nu^2)$.
- Let $\chi : F^\times\backslash\mathbb{A}^\times \to \mathbb{C}^\times$ be a unitary idele class character.
- Let

$$f(g,s) = f(g,s;\Phi,\chi) = |\det(g)|^s \int_{\mathbb{A}^\times} \Phi(ae_2g)\chi(a)|a|^{2s} d^\times a,$$

  where $e_2 = (0,1) \in F^2$. Then

$$f\left(\left(\begin{smallmatrix} a & b \\ 0 & d \end{smallmatrix}\right) g, s\right) = \left|\frac{a}{d}\right|^s \chi(d)^{-1} f(g,s).$$

  I.e., $f \in \mathrm{Ind}_{B_2(\mathbb{A})}^{\mathrm{GL}_2(\mathbb{A})}(\delta_{B_2}^{s-\frac{1}{2}}\chi^{-1})$, where δ_{B_2} is the modulus character: $\delta_{B_2}\left(\begin{smallmatrix} a & b \\ 0 & d \end{smallmatrix}\right) = \left|\frac{a}{d}\right|$, and χ is extended to B_2 by $\chi\left(\begin{smallmatrix} a & b \\ 0 & d \end{smallmatrix}\right) = \chi(d)$.
- Define the Eisenstein series

$$E(g,s) = E(g,s;\Phi,\chi) = \sum_{\gamma \in B_2(F)\backslash \mathrm{GL}_2(F)} f(\gamma g, s).$$

An Eulerian integral for $\mathrm{GL}_2 \times \mathrm{GL}_2$

- Let $(\pi_1, V_{\pi_1}), (\pi_2, V_{\pi_2})$ be irreducible automorphic cuspidal representations of $\mathrm{GL}_2(\mathbb{A})$. We have tensor product decomposition $\pi_i = \otimes'_\nu \pi_{i,\nu}$ into a restricted tensor product of local irreducible admissible representations. Denote their central character by ω_{π_i}.
- For $\varphi_i \in V_{\pi_i}$, $i \in \{1, 2\}$, non-zero cusp forms, we form the integral

$$I(s, \varphi_1, \varphi_2, \Phi) := \int_{Z(\mathbb{A})\mathrm{GL}_2(F)\backslash \mathrm{GL}_2(\mathbb{A})} \varphi_1(g)\varphi_2(g)E(g, s; \Phi, \omega_{\pi_1}\omega_{\pi_2})dg.$$

- Unfolding the Eisenstein series and replacing φ_1 by its Fourier expansion

$$\varphi_1(g) = \sum_{\gamma \in F^\times} W_{\varphi_1,\psi}\left(\begin{pmatrix} \gamma & \\ & 1 \end{pmatrix} g\right),$$

where $W_{\varphi_1,\psi}$ is the ψ-Whittaker function of φ_1 given by

$$W_{\varphi_1,\psi}(g) = \int_{F\backslash \mathbb{A}} \varphi_1\left(\begin{pmatrix} 1 & x \\ & 1 \end{pmatrix} g\right)\psi^{-1}(x)dx,$$

we get $I(s, \varphi_1, \varphi_2, \Phi) = \Psi(s, W_{\varphi_1,\psi}, W_{\varphi_2,\psi^{-1}}, \Phi)$ where

$$\Psi(s, W_{\varphi_1,\psi}, W_{\varphi_2,\psi^{-1}}, \Phi) = \int_{N_2(\mathbb{A})\backslash \mathrm{GL}_2(\mathbb{A})} W_{\varphi_1,\psi}(g)W_{\varphi_2,\psi^{-1}}(g)\Phi(e_2 g)|\det(g)|^s dg.$$

An Eulerian integral for $\mathrm{GL}_2 \times \mathrm{GL}_2$ continued

- The integral unfolds to a unique model. Since $\pi_1 \cong \otimes' \pi_{1,v}$, if $\varphi_1 \in V_{\pi_1}$ corresponds to $\otimes_\nu \xi_{1,\nu}$, then by the uniqueness of the Whittaker model, we have

$$W_{\varphi_1,\psi}(g) = \prod_\nu W_{\xi_{1,\nu},\psi_\nu}(g_\nu).$$

- Thus, if φ_1, φ_2 and Φ are all decomposable, then

$$I(s, \varphi_1, \varphi_2, \Phi) = \prod_\nu \Phi_\nu(s, W_{\xi_{1,\nu},\psi_\nu}, W_{\xi_{2,\nu},\psi_\nu^{-1}}, \Phi_\nu), \quad \text{for } \operatorname{Re}(s) > 1,$$

  where the local integrals $\Phi_\nu(s, W_{\xi_{1,\nu},\psi_\nu}, W_{\xi_{2,\nu},\psi_\nu^{-1}}, \Phi_\nu)$ are given by

$$\int_{N_2(F_\nu)\backslash \mathrm{GL}_2(F_\nu)} W_{\xi_{1,\nu},\psi_\nu}(g_\nu) W_{\xi_{2,\nu},\psi_\nu^{-1}}(g_\nu) \Phi_\nu(e_2 g_\nu) |\det g_\nu|_\nu^s \, dg_\nu.$$

- Then one can compute the local unramified integral explicitly to get the local L-function.
- Until the late 1980's, all known Rankin-Selberg integrals that represent an L-function were shown to unfold to a unique model (or involve certain uniqueness result).

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New Way integral of Piatetski-Shapiro and Rallis

- In 1988, Piatetski-Shapiro and Rallis constructed a global integral which unfolds to a non-unique model, and represents the standard L-function (or its twist by a character) for Sp_{2n}. This is the first example of an integral representation which unfolds to a non-unique model. Their method is known as the New Way method (named after the title of [PSR1988]: "A new way to get Euler products").
- Their integral is defined for any irreducible, automorphic, cuspidal representation of $\mathrm{Sp}_{2n}(\mathbb{A})$. In particular, the representation is not required to have certain unique model such as the Whittaker model.

Known New Way constructions

Non-unique models are ubiquitous in the theory of automorphic representations, but only a handful of them are used to represent an L-function.

- Soudry (1988): degree five L-function for GSp_4
- Bump-Furusawa-Ginzburg (1995): standard L-function for classical groups
- Qin (2007): standard L-function for quasi-split unitary groups
- Gurevich-Segal (2015): standard L-function for G_2
- Segal (2017): standard L-function for $G_2 \times \mathrm{GL}_1$
- Pollack-Shah (2017): Spin L-function for GSp_4
- Pollack-Shah (2018): Spin L-function for GSp_6

- Bump-Friedberg (1999), Ginzburg (2018): n-fold metaplectic covering group $\mathrm{GL}_r^{(n)}$.

Question: How to extend the construction to tensor product L-function?

Recent work of Ginzburg and Soudry

- In 2020, Ginzburg and Soudry re-considered the Piatetski-Shapiro and Rallis's **New Way integral**, and showed that one can derive the **New Way integral** (before unfolding) from the **doubling integral** (after unfolding), through global computations involving
  1. global root exchange,
  2. identities between Eisenstein series.
- The **doubling integral** of Piatetski-Shapiro and Rallis (1987) has been generalized by Cai, Friedberg, Ginzburg, Kaplan (2019) to represent the tensor product L-function for $G \times \mathrm{GL}_n$ (**generalized/twisted doubling method**).
- Ginzburg and Soudry applied a similar computation to the **twisted doubling integral** (after unfolding) for $\mathrm{Sp}_4 \times \mathrm{GL}_2$ to derive an explicit global integral, and they conjectured that this integral is Eulerian by the New Way method and represents the tensor product partial L-function for $\mathrm{Sp}_4 \times \mathrm{GL}_2$.

Notation

- F a number field, $\mathbb{A}$ its ring of adeles, ψ non-trivial additive character of $F\backslash\mathbb{A}$. We let $[G] = G(F)\backslash G(\mathbb{A})$.
- $J_2 = \left(\begin{smallmatrix} 0 & 1 \\ 1 & 0 \end{smallmatrix}\right) \in \mathrm{GL}_2$
- $Sp_4 = \{g \in \mathrm{GL}_4 : {}^t g \begin{pmatrix} & J_2 \\ -J_2 & \end{pmatrix} g = \begin{pmatrix} & J_2 \\ -J_2 & \end{pmatrix}\}$

More notation

- $SO_{T_0} = \{g \in \mathrm{SL}_2 : {}^t g T_0 g = T_0\}$, where $T_0 \in \mathrm{Sym}_2(F) \cap \mathrm{GL}_2(F)$
- $T = J_2 T_0$
- χ_T the quadratic character on $F^\times \backslash \mathbb{A}^\times$, given by $\chi_T(x) = (x, \det(T))$ where $(\cdot,\cdot)$ is the global Hilbert symbol
- Consider the dual pair $\mathrm{SO}_{T_0} \times \mathrm{Sp}_4$ inside Sp_8. The adelic group $\mathrm{SO}_{T_0}(\mathbb{A}) \times \mathrm{Sp}_4(\mathbb{A})$ splits in $\widetilde{\mathrm{Sp}}_8(\mathbb{A})$, so let i_T be a splitting. We consider the restriction of the Weil representation ω_ψ of $\widetilde{\mathrm{Sp}}_8(\mathbb{A})$, corresponding to the character ψ, to the group $\mathrm{SO}_{T_0}(\mathbb{A}) \times \mathrm{Sp}_4(\mathbb{A})$ under the splitting.
- $N_{2,8} = \left\{ \begin{pmatrix} I_2 & x & y & z \\ & I_2 & & y^* \\ & & I_2 & x^* \\ & & & I_2 \end{pmatrix} \in \mathrm{Sp}_8 \right\}$ has a structure of the Heisenberg group $\mathcal{H}_9$ in 9 variables via the map
$$\alpha_T \begin{pmatrix} I_2 & x & y & z \\ & I_2 & & y^* \\ & & I_2 & x^* \\ & & & I_2 \end{pmatrix} = (x, y, \mathrm{tr}(Tz)).$$
- Realize the Weil representation in the Schwartz space $\mathcal{S}(\mathrm{Mat}_2(\mathbb{A}))$. For $\Phi \in \mathcal{S}(\mathrm{Mat}_2(\mathbb{A}))$, $v \in N_{2,8}(\mathbb{A})$, $(m,h) \in \mathrm{SO}_{T_0}(\mathbb{A}) \times \mathrm{Sp}_4(\mathbb{A})$, form the theta series
$$\theta_\psi^\Phi(\alpha_T(v)(m,h)) := \theta_\psi^\Phi(\alpha_T(v) i_T(m,h)) = \sum_{x \in \mathrm{Mat}_2(F)} \omega_\psi(\alpha_T(v) i_T(m,h))\Phi(x).$$

More notation II

- We embed $\mathrm{SO}_{T_0} \times \mathrm{Sp}_4$ inside Sp_8 via $(m, h) \mapsto \mathrm{diag}(m, h, m^*)$ where $m^* = J_2{}^t m^{-1} J_2$. We re-denote $(m, h) = \mathrm{diag}(m, h, m^*)$.
- π an irreducible automorphic cuspidal representation of $\mathrm{Sp}_4(\mathbb{A})$.
- $\varphi \in V_\pi$ a non-zero cusp form.
- Let τ be an irreducible unitary cuspidal automorphic representation of $\mathrm{GL}_2(\mathbb{A})$.
- Let $f_{\Delta(\tau\otimes\chi_T,2),s} \in \mathrm{Ind}_{P_8(\mathbb{A})}^{\mathrm{Sp}_8(\mathbb{A})}(\Delta(\tau \otimes \chi_T, 2)|\det\cdot|^s)$ where $\Delta(\tau \otimes \chi_T, 2)$ is the generalized Speh representation of $\mathrm{GL}_4(\mathbb{A})$ attached to $\tau \otimes \chi_T$. Here $P_8 = M_8 \ltimes N_8$ is the Siegel parabolic subgroup of Sp_8, with Levi $M_8 \cong \mathrm{GL}_4$. Let $E(\cdot, f_{\Delta(\tau\otimes\chi_T,2),s})$ be the corresponding Eisenstein series on $\mathrm{Sp}_8(\mathbb{A})$ given by

$$E(g, f_{\Delta(\tau\otimes\chi_T,2),s}) = \sum_{\gamma\in P_8(F)\backslash \mathrm{Sp}_8(F)} f_{\Delta(\tau\otimes\chi_T,2),s}(\gamma g).$$

A Conjecture of Ginzburg and Soudry

Conjecture (Ginzburg-Soudry, 2020)

Let S be certain finite set of places. The integral

$$\mathcal{Z}(\varphi, \theta_\psi^\Phi, E(\cdot, f_{\Delta(\tau\otimes\chi_T,2),s})) = \int\limits_{[\mathrm{Sp}_4]} \int\limits_{[N_{2,8}]} \varphi(h)\theta_\psi^\Phi(\alpha_T(v)(1,h))E(v(1,h), f_{\Delta(\tau\otimes\chi_T,2),s})dvdh$$

is Eulerian in the sense of the New Way method, and represents the partial L-function $L^S(s+\frac{1}{2}, \pi\times\tau)$.

- The above conjecture made by Ginzburg and Soudry is the first example of a New Way type integral representation for the L-function for $G \times \mathrm{GL}_k$, where $k > 1$.

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Main Theorem

We take the normalized Eisenstein series

$$E^{*,S}(\cdot, f_{\Delta(\tau\otimes\chi_T,2),s}) = d^{\mathrm{Sp}_8,S}_{\tau\otimes\chi_T}(s)E(\cdot, f_{\Delta(\tau\otimes\chi_T,2),s}),$$

where $d^{\mathrm{Sp}_8,S}_{\tau\otimes\chi_T}(s) = \prod_{\nu\notin S} d^{\mathrm{Sp}_8}_{\tau_\nu\otimes\chi_T}(s)$, and

$$d^{\mathrm{Sp}_8}_{\tau_\nu\otimes\chi_T}(s) = L(s+\frac{3}{2}, \tau_\nu\otimes\chi_T)L(2s+2, \tau_\nu\otimes\chi_T, \mathrm{Ext}^2)L(2s+1, \tau_\nu\otimes\chi_T, \mathrm{Sym}^2).$$

Theorem (Y.)

The conjecture of Ginzburg-Soudry holds. That is, there is a choice of data so that

$$\mathcal{Z}(\varphi, \theta^\Phi_\psi, E^{*,S}(\cdot, f_{\Delta(\tau\otimes\chi_T,2),s})) = L^S(s+\frac{1}{2}, \pi\times\tau)\cdot \mathcal{Z}_S(\varphi, \Phi, f^*_{\mathcal{W}(\tau\otimes\chi_T,2,\psi_{2T}),s}),$$

where $\mathcal{Z}_S(\varphi, \Phi, f^*_{\mathcal{W}(\tau\otimes\chi_T,2,\psi_{2T}),s})$ *is a meromorphic function. Moreover, for any* $s_0\in\mathbb{C}$*, the data can be chosen so that* $\mathcal{Z}_S(\varphi, \Phi, f^*_{\mathcal{W}(\tau\otimes\chi_T,2,\psi_{2T}),s})$ *is non-zero at* s_0*.*

Main steps of my proof

The proof roughly consists of three major steps:

1. Global unfolding
2. Local computation (unramified, finite ramified, and archimedean)
3. Global computation

Step 1: Unfolding

- Recall the integral

$$\mathcal{Z}(\varphi, \theta_\psi^\Phi, E^{*,S}(\cdot, f_{\Delta(\tau\otimes\chi_T,2),s})) = \int_{[\mathrm{Sp}_4]}\int_{[N_{2,8}]} \varphi(h)\theta_\psi^\Phi(\alpha_T(v)(1,h))E^{*,S}(v(1,h), f_{\Delta(\tau\otimes\chi_T,2),s})dvdh.$$

- Let

$$\gamma = \begin{pmatrix} & I_2 & & \\ & & & -I_2 \\ I_2 & & & \\ & & I_2 & \end{pmatrix}, N_{2,8}^0 = \left\{ v(x,0,z) = \begin{pmatrix} I_2 & x & 0 & z \\ & I_2 & 0 & 0 \\ & & I_2 & x^* \\ & & & I_2 \end{pmatrix} \in N_{2,8} \right\}.$$

Theorem (Y.)

The integral $\mathcal{Z}(\varphi, \theta_\psi^\Phi, E^{*,S}(\cdot, f_{\Delta(\tau\otimes\chi_T,2),s}))$ converges absolutely when $Re(s) \gg 0$ and can be meromorphically continued to all $s \in \mathbb{C}$. When $Re(s) \gg 0$, the integral $\mathcal{Z}(\varphi, \theta_\psi^\Phi, E^{*,S}(\cdot, f_{\Delta(\tau\otimes\chi_T,2),s}))$ unfolds to

$$\int_{N_4(\mathbb{A})\backslash Sp_4(\mathbb{A})}\int_{N_{2,8}^0(\mathbb{A})} \varphi_{\psi,T}(h)\omega_\psi(\alpha_T(v)(1,h)))\Phi(I_2) f^*_{\mathcal{W}(\tau\otimes\chi_T,2,\psi_{2T}),s}(\gamma v(1,h))dvdh.$$

Unfolding continued

- Here, $\varphi_{\psi,T}$ is the Fourier coefficient

$$\varphi_{\psi,T}(g) = \int_{[N_4]} \varphi(ng)\psi_T(n)dn, \ \ N_4 = \{\begin{pmatrix} I_2 & z \\ & I_2 \end{pmatrix} \in \mathrm{Sp}_4\},$$

ψ_T is given by

$$\psi_T \begin{pmatrix} I_2 & z \\ & I_2 \end{pmatrix} = \psi(\mathrm{tr}(Tz)),$$

and

$$f^*_{\mathcal{W}(\tau\otimes\chi_T,2,\psi_{2T}),s}(g) = d^{\mathrm{Sp}_8,S}_{\tau\otimes\chi_T}(s) f_{\mathcal{W}(\tau\otimes\chi_T,2,\psi_{2T}),s}(g),$$

where $f_{\mathcal{W}(\tau\otimes\chi_T,2,\psi_{2T}),s}$ is the composition of the section and the unique functional attached to $\Delta(\tau \otimes \chi_T, 2)$; for any $g \in \mathrm{Sp}_8(\mathbb{A})$,

$$f_{\mathcal{W}(\tau\otimes\chi_T,2,\psi_{2T}),s}(g) = \int\limits_{U_{(2^2)}(F)\backslash U_{(2^2)}(\mathbb{A})} f_{\Delta(\tau\otimes\chi_T,2),s}\left(\left(\begin{smallmatrix} u & \\ & u^* \end{smallmatrix}\right) g\right) \psi^{-1}_{2T}(u)du,$$

where $U_{(2^2)} = \left\{\left(\begin{smallmatrix} I_2 & x \\ & I_2 \end{smallmatrix}\right) \in \mathrm{GL}_4\right\}$, and $\psi_{2T}\left(\begin{smallmatrix} I_2 & x \\ & I_2 \end{smallmatrix}\right) = \psi\left(\mathrm{tr}(2Tx)\right)$.

Non-unique model

- Key observations of Fourier coefficient $\varphi_{\psi,T}$:
  1. By a Theorem of Jian-Shu Li (1992), every non-zero cusp form affords a non-zero Fourier coefficient $\varphi_{\psi,T}$ for some non-singular matrix T.
  2. $\varphi_{\psi,T}$ does not correspond to a unique model. So $\varphi_{\psi,T}$ does not factorize as an Euler product. (Everything else in the integral is factorizable)
- The most important ingredient of the New Way method is the local unramified computation: the local integral with unramified data produces the local L-function for any functional with the same invariance properties applied to a spherical vector.

Step 2: Unramified computation

- Let F_ν be a non-archimedean local field, $\mathcal{O}_{F_\nu}$ its ring of integers. Assume all entries of T are in $\mathcal{O}_{F_\nu}^\times$.
- Suppose π_ν is an irreducible unramified representation of $\mathrm{Sp}_4(F_\nu)$. Let $v_0 \in V_{\pi_\nu}$ be a non-zero unramified vector in V_{π_ν}.
- Suppose τ_ν is an irreducible unramified generic representation of $\mathrm{GL}_2(F_\nu)$
- $\Phi_\nu^0 = \mathbf{1}_{\mathrm{Mat}_2(\mathcal{O}_{F_\nu})}$ is the characteristic function of $\mathrm{Mat}_2(\mathcal{O}_{F_\nu})$
- $f^*_{\mathcal{W}(\tau_\nu \otimes \chi_T, 2, \psi_{2T}), s}$ is an unramified section in $\mathrm{Ind}_{P_8(F_\nu)}^{\mathrm{Sp}_8(F_\nu)}(\mathcal{W}(\tau_\nu \otimes \chi_T, 2, \psi_{2T})|\det|^s)$ appropriately normalized
- Let $l_T : V_{\pi_\nu} \to \mathbb{C}$ be a linear functional on V_{π_ν} such that

$$l_T\left(\pi_\nu\begin{pmatrix} I_2 & z \\ & I_2 \end{pmatrix} v\right) = \psi^{-1}(\mathrm{tr}(Tz))l_T(v), \quad \text{for all } v \in V_{\pi_\nu}. \qquad (1)$$

Unramified computation continued

- Denote

$$\mathcal{Z}_\nu^*(l_T, s) = \int\limits_{N_4(F_\nu)\backslash \mathrm{Sp}_4(F_\nu)} \int\limits_{N_{2,8}^0(F_\nu)} l_T(\pi_\nu(h)v_0)$$
$$\omega_{\psi,\nu}(\alpha_T(v)(1,h))\Phi_\nu^0(l_2) f^*_{\mathcal{W}(\tau_\nu \otimes \chi_T, 2, \psi_{2T}), s}(\gamma v(1,h))dvdh.$$

Theorem (Y.)

For Re(s) $\gg 0$ and for any linear functional l_T satisfying (1), we have

$$\mathcal{Z}_\nu^*(l_T, s) = L(s + \frac{1}{2}, \pi_\nu \times \tau_\nu) \cdot l_T(v_0).$$

The proof of above Theorem uses the local unramified integral for $L(s + \frac{1}{2}, \pi_\nu \times \tau_\nu)$ from the twisted doubling method of Cai-Friedberg-Ginzburg-Kaplan (2019).

Step 3: Global computation

- Now we follow Piatetski-Shapiro and Rallis's method.
- For a finite set Ω of places, denote

$$\mathcal{Z}_\Omega(\varphi, \Phi, f^*_{\mathcal{W}(\tau\otimes\chi_T,2,\psi_{2T}),s}) = \int_{N_4(\mathbb{A}_\Omega)\backslash \mathrm{Sp}_4(\mathbb{A}_\Omega)} \int_{N^0_{2,8}(\mathbb{A}_\Omega)}$$

$$\varphi_{\psi,T}(h)\omega_{\psi,\Omega}\left(\alpha_T(v)(1,h)\right))\Phi_\Omega(I_2) f^*_{\mathcal{W}(\tau_\Omega\otimes\chi_T,2,\psi_{2T}),s}(\gamma v(1,h))dvdh,$$

where $\mathrm{Sp}_4(\mathbb{A}_\Omega) = \prod_{\nu\in\Omega} \mathrm{Sp}_4(F_\nu)$, etc.

- Using the unramified computation, we can show that if Ω is a finite set of places containing S and $\nu \notin \Omega$, then

$$\mathcal{Z}_{\Omega\cup\{\nu\}}(\varphi, \Phi, f^*_{\mathcal{W}(\tau\otimes\chi_T,2,\psi_{2T}),s}) = L(s+\frac{1}{2}, \pi_\nu \times \tau_\nu)\mathcal{Z}_\Omega(\varphi, \Phi, f^*_{\mathcal{W}(\tau\otimes\chi_T,2,\psi_{2T}),s}).$$

- Finally we take the limit to obtain the result, concluding the proof of the Main Theorem.

Thank you for your attention!