Quantum ergodicity in the level aspect on higher rank real and p-adic locally symmetric spaces

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Geodesic flow on hyperbolic surface

- Y = compact hyperbolic surface
- $\Phi_t \curvearrowright T^1 Y$ geodesic flow

curvature $< 0 \implies \Phi_t$ is *ergodic*

$\implies$ generic geodesics *equidistribute*

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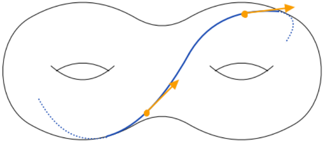

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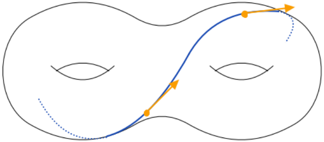

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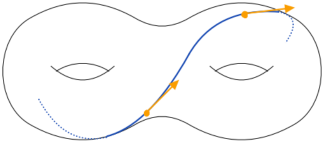

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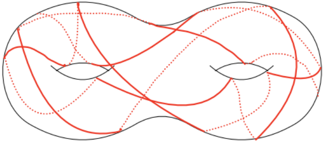

Classical to quantum

- Classical particle $\rightsquigarrow$ point $p = (y, v)$ in T^1Y
- Evolution of $p \rightsquigarrow$ apply geodesic flow
- Quantum particle $\rightsquigarrow \psi \in L^2(Y)$
- Evolution of $\psi \rightsquigarrow$ solve Schrödinger equation $\rightsquigarrow$ eigenfunctions of Δ
- High energy quantum particle $\rightsquigarrow$ eigenfunction of Δ with large eigenvalue

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Quantum ergodicity

- Born rule in quantum mechanics:

$$\mathbb{P}(\text{observing } \psi \text{ in } E \subset Y) = \int_Y \mathbb{1}_E \cdot |\psi|^2 \, đ\mathrm{Vol}$$

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$$\mathbb{P}(\text{observing } \psi \text{ in } E \subset Y) = \frac{\mathrm{Vol}(E)}{\mathrm{Vol}(Y)} = \int_Y \mathbb{1}_E \, đ\mathrm{Vol}$$

- Quantum ergodicity essentially says that for all $E \subset Y$ we have

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- Let Y_n be a sequence of hyperbolic surfaces "converging" to hyperbolic plane (e.g. tower of congruence covers of arithmetic hyperbolic surface).
- Let $(\lambda_j^{(n)}, \psi_j^{(n)})$ be the eigendata of Y_n.
- Let $\mathcal{I} \subset (\frac{1}{4}, \infty)$ be a compact interval.
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Real and p-adic (locally) symmetric spaces

	rank one	higher rank
archimedean	hyperbolic surfaces $SL(2, \mathbb{R})$	symmetric spaces $SL(n, \mathbb{R})$, etc.
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Symmetric spaces

- G = semisimple Lie group over $\mathbb{R}$ (w/o compact factors)
- X is associated Riemannian manifold called *symmetric space*
- $X = G/K$ with K a maximal compact subgroup
- $D(G, K)$ = G-invariant differential operators on X
- Fact: $D(G, K)$ generated by k operators

$G = \mathrm{SL}(2, \mathbb{R})$

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Figure: Δ closely related to averaging over spheres in $\mathbb{H}$

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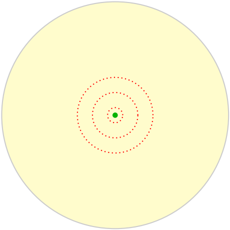

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Bruhat-Tits buildings

- G = semisimple algebraic group over F (non-archimedean local field)
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$$G = \mathrm{PGL}(2, \mathbb{Q}_p)$$
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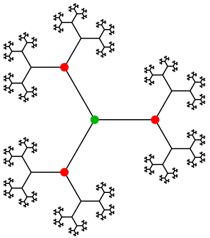

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Buildings are composed of branching apartments

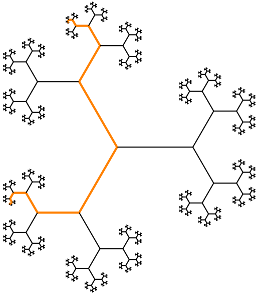
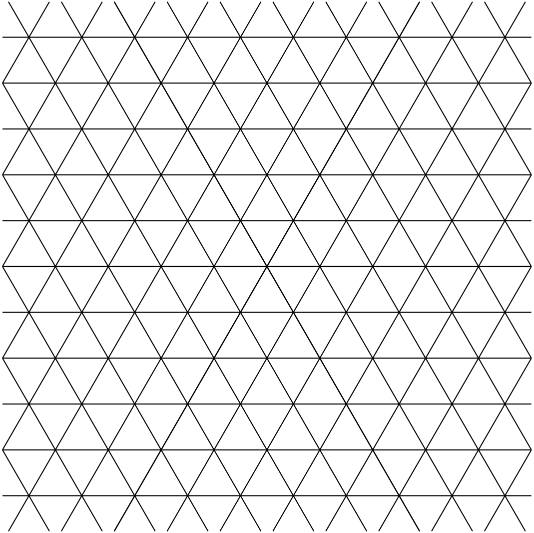
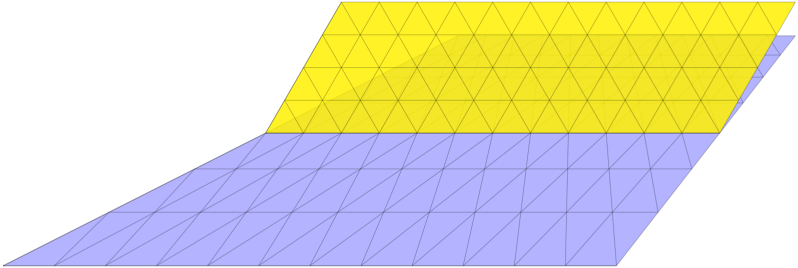


Figure: An apartment in the tree is a bi-infinite geodesic.

An apartment in the Bruhat-Tits building of $\mathrm{PGL}(3, \mathbb{Q}_p)$



Branching apartments



Visualization of building

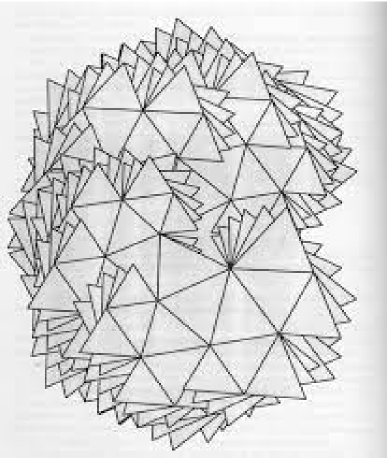
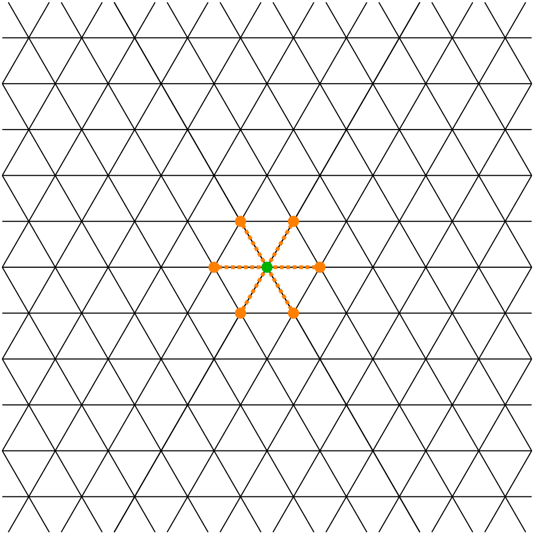


Figure: Picture made by Paul Garrett

Spherical Hecke algebra as refinements of adjacency operator



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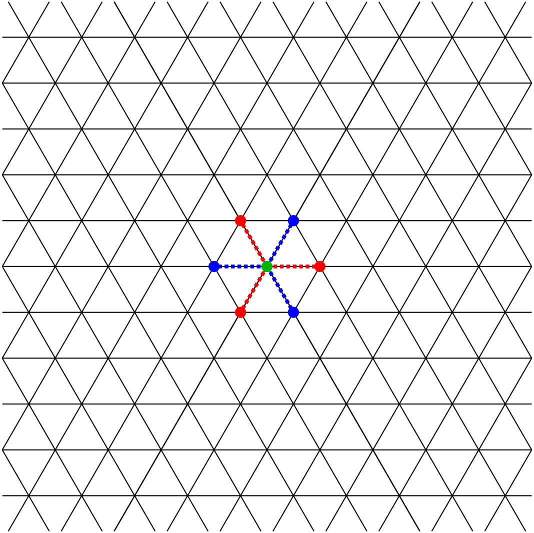

Quotients of symmetric spaces and buildings

- Let $X = G/K =$ symmetric space or Bruhat-Tits building
- Let Γ be a cocompact lattice in G (e.g. uniform arithmetic lattice)

$$X/\Gamma \text{ is } \begin{cases} \text{locally symmetric space (e.g. hyperbolic surface)} \\ \text{finite simplicial complex (e.g. finite regular graph)} \end{cases}$$

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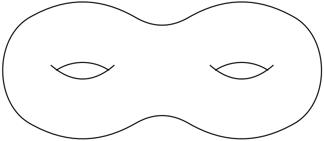

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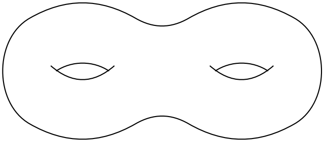

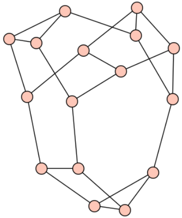

Quantization of ergodic flow on X/Γ

	rank one	higher rank
archimedean	geod. flow ($\mathbb{R}$-action) on hyperbolic surface $\downarrow$ Laplacian	Weyl chamber flow ($\mathbb{R}^k$-action) on loc. symmetric space $\downarrow$ invariant differential ops
non-archimedean	geod. flow ($\mathbb{Z}$-action) on regular graph $\downarrow$ adjacency op.	Weyl chamber flow ($\mathbb{Z}^k$-action) on p-adic loc. symmetric space $\downarrow$ spherical Hecke algebra

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Representation theory

- Let $\mathcal{C} =$ either $D(G, K)$ (symmetric space) or $H(G, K)$ (Bruhat-Tits building)

- Then $\mathcal{C} \curvearrowright L^2(\Gamma \backslash G / K)$

joint eigenfn of $\mathcal{C} \leftrightarrow K$-fixed vector in irrep of $L^2(\Gamma \backslash G)$ (e.g. Maass form)

joint eigenvals of $\mathcal{C} \leftrightarrow$ infinitesimal character/Satake parameters of the irrep

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Cartoon version of main theorem

Let $\mathcal{B}$ be the Bruhat-Tits building associated to $\mathrm{PGL}(3, F)$ with F a non-archimedean local field of arbitrary characteristic.

Let Y_n be a sequence of compact quotients of $\mathcal{B}$ which Benjamini-Schramm converges to $\mathcal{B}$ (e.g. injectivity radius $\to \infty$).

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Exact statement of main theorem

Let $G = \mathrm{PGL}(3, F)$ and $K = \mathrm{PGL}(3, \mathcal{O})$, where F is a non-archimedean local field of arbitrary characteristic and $\mathcal{O}$ is its ring of integers.

Suppose $Y_n = \Gamma_n \backslash G / K$ with Γ_n torsionfree lattices such that $\mathrm{vol}(Y_n) \to \infty$.

For each Y_n let $\{\psi_j^{(n)}\}$ be an ONB of eigenfunctions of $H(G, K) \curvearrowright L^2(Y_n)$ with Satake parameters $\nu_j^{(n)}$. Let Θ be a subset of the tempered spectrum which has positive Plancherel measure, $\partial\Theta$ has zero Plancherel measure, and $\mathrm{cl}(\Theta)$ does not meet a codimension one exceptional locus Ξ (e.g. where two Satake parameters are equal). Let $a_n \in L^\infty(Y_n)$ with uniform L^∞-bound and orthogonal to coloring eigenfunctions (coming from 1D spherical irreps in $L^2(\Gamma_n \backslash G)$). Then

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